Thiết kế bộ điều khiển tỉ lệ dựa trên thụ động cho hệ thống động

A passivity approach to proportional controller design for dynamic systems

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Tóm tắt nội dung
Báo báo này liên quan đến các hệ thống động mà động lực là *affine* theo đầu vào điều khiển. Động lực của hệ được xem xét để viết lại trong một dạng chính tắc, gọi là biểu diễn Hamilton thụ động, nhằm giúp khảo sát rõ hơn các đặc tính cấu trúc và thiết kế điều khiển (như ma trận kết nối/giảm chấn, hàm trữ năng bị chặn và thiết kế bộ điều khiển tỉ lệ). Hệ khối lượng lò xo giảm xóc được sử dụng để minh họa phương pháp. Mô phỏng số được thực hiện trong vòng hở và vòng kín.

Từ khóa: Biểu diễn Hamilton, mô hình hóa, hệ động lực, thụ động, bộ điều khiển tỉ lệ.

Abstract
This work concerns dynamical systems whose dynamics are affine in the control input. Such dynamics are considered to write into a canonical form, namely the passive port-Hamiltonian representation in order to explore further some structural properties usable for the control design (such as interconnection and damping matrices, bounded Hamiltonian storage function and proportional feedback controller design). The case of a mass-spring-damper system is used to illustrate the approach. Besides, numerical simulations are included in both the open loop and closed loop.

Keywords: Port-Hamiltonian representation, modeling, dynamical systems, passivity, proportional controller.

1. INTRODUCTION

This paper deals with the port-based modeling of general nonlinear dynamical systems Khalil (2002); Ortega *et al.* (1998); Van der Schaft (2017); Brogliato *et al.* (2007) whose dynamics are described by a set of Ordinary Differential Equations (ODEs) and affine in the input u as follows :

$$\dot{x} = f(x) + g(x)u, \quad x(t=0) = x_{init} \tag{1}$$

where $x = x(t)$ is the state vector in the operating region $\mathbb{D} \in \mathbb{R}^n$, $f(x) \in \mathbb{R}^n$ expresses the smooth (nonlinear) function with respect to the vector field x. The input-state map and the control input are represented by $g(x) \in \mathbb{R}^{n\times m}$ and $u \in \mathbb{R}^m$, respectively. It is worth noting that many industrial applications occurred in electrical systems, electromechanical systems or biochemical systems, etc. belong to this kind of systems Maschke *et al.* (2000); Van der Schaft (2000); Ortega *et al.* (2001, 2002); Antonelli and Astolfi (2003); Favache and Dochain (2010); Ramírez *et al.* (2013); Guay and Hudon (2016).

In addition to the Bond graph modeling [1] Couenne *et al.* (2006); Eberard *et al.* (2007); Vu *et al.* (2016), the port-based modeling Maschke *et al.* (2000); Van der Schaft (2000) leads to extension of the so-called port-Hamiltonian (pH) systems. It is always important to transfer the (original) dynamics of the systems to the port Hamil-

⋆ The material in this paper was partially presented at the 4th Conference on Electrical, Electronics, Communications and Automation (EECA), October 28, 2017, Duy Tân University, Đà Nẵng, Việt Nam.

[1] In general, the Bond graph representation of physical systems combines the effort variables with the flow variables (or the generalized efforts with the generalized flows) through junctions.

tonian representation prior to developing state feedback laws for control Ortega *et al.* (2002, 2008); Ortega and Borja (2014); Sira-Ramírez and Angulo-Núñez (1997); Sira-Ramírez (1998). In other words, once a canonical form (i.e., expressed by the pH model Maschke *et al.* (2000); Van der Schaft (2000); Ortega *et al.* (2001)) of the system dynamics is *a priori* derived, then the passivity-based control strategy or interconnection and damping assignment passivity-based control (IDA-PBC) and other extensions (such as the energy shaping control or tracking-error-based control) can be advantageously applied to show stabilization properties. The proportional feedback law design and control scenarios proposed for the simulations are main contributions of this work.

This paper is organized as follows. Section 2 gives a brief overview of the pH representation of (affine) nonlinear dynamical systems. Section 3 is devoted to the modelling and control design of a mass-spring-damper system in the pH framework. Some further discussions are given in Section 4. Section 5 ends the paper with some concluding remarks.

Notations: The following notations are considered throughout the paper :

- $\mathbb{R}$ is the the set of real number.
- $\top$ is the matrix transpose operator.
- m and n ($m \leq n$) are the positive integers.
- x_{init} is the initial value of the state vector.

2. THE PASSIVE PORT-HAMILTONIAN REPRESENTATION OF AFFINE DYNAMICAL SYSTEMS

Assume that if the function $f(x)$ verifies the so-called separability condition Guay and Hudon (2016); Dörfler *et al.* (2009); Ramírez *et al.* (2009); Favache *et al.* (2011); Hudon *et al.* (2015); Hoang *et al.* (2017), that is, $f(x)$ can be decomposed and expressed as the product of some (interconnection and damping) structure matrices and the gradient of a potential function with respect to the state variables, i.e., the co-state variables :

$$f(x) = [J(x) - R(x)] \frac{\partial \mathcal{H}(x)}{\partial x} \tag{2}$$

where $J(x)$ and $R(x)$ are the $n \times n$ skew-symmetric interconnection matrix (i.e., $J(x) = -J^\top(x)$) and the $n \times n$ symmetric damping matrix (i.e. $R(x) = R^\top(x)$), respectively while $\mathcal{H}(x) : \mathbb{R}^n \longrightarrow \mathbb{R}$ represents the Hamiltonian storage function of the system (possibly related to the total energy of the system). Furthermore, if the damping matrix $R(x)$ is positive semi-definite,

$$R(x) \geq 0 \tag{3}$$

Then, the original dynamics described by (1) is said to be a pH representation with dissipation Maschke *et al.* (2000); Van der Schaft (2000); Ortega *et al.* (2001). Equation (1) is then rewritten as follows :

$$\begin{cases} \dot{x} = [J(x) - R(x)] \dfrac{\partial \mathcal{H}(x)}{\partial x} + g(x)u \\ y = g(x)^\top \dfrac{\partial \mathcal{H}}{\partial x} \end{cases} \tag{4}$$

where y is the output.

It can be clearly seen for the pH models defined by (3)(4) that the time derivative of the Hamiltonian storage function $\mathcal{H}(x)$ satisfies the energy balance equation Ortega *et al.* (2001) below

$$\frac{d\mathcal{H}(x)}{dt} = - \left[\frac{\partial \mathcal{H}(x)}{\partial x} \right]^\top R(x) \frac{\partial \mathcal{H}(x)}{\partial x} + u^\top y \tag{5}$$

Thanks to (3), (5) becomes :

$$\underbrace{\frac{d\mathcal{H}(x)}{dt}}_{\text{stored power}} \leq \underbrace{u^\top y}_{\text{supplied power}} \tag{6}$$

From a physical point of view, inequality in (6) implies that the total amount of energy supplied from external source is always greater than the increase in the energy stored in the system. Also, equality in (6) holds only if the damping matrix $R(x)$, that is strongly related to the dissipation term, is equal to 0. Hence, the pH system (4) is said to be passive with input u and output y corresponding to the Hamiltonian storage function $\mathcal{H}(x)$ Van der Schaft (2017); Bao and Lee (2007). This is one of advantageous features of the pH representation and has been applied for the control design, even for the stabilization of infinite dimensional systems (see e.g., Alonso and Ydstie (2001); Hoang and Phan (2016)).

We shall not elaborate any further on the pH representation here (for example, the concepts related to the cyclo-passive/passive property or Dirac structure, etc.) and refer the reader to Maschke *et al.* (2000); Van der Schaft (2000); Ortega *et al.* (2002); Dörfler *et al.* (2009); Hoang *et al.* (2017) for more information.

3. THE MASS-SPRING-DAMPER SYSTEM CASE STUDY

To illustrate the concepts proposed in Section 2, we illustrate our main points with a simple case study, that is the mass-spring-damper system. Originally, the port-Hamiltonian representation has been first considered for electrical or mechanical systems as seen in the literature (see e.g., (Ortega *et al.* (1998); Van der Schaft (2000); Batlle (2005))).

A car and its suspension system traveling over a bumpy road can be modeled as a mass-spring-damper system as shown in Figure 1 Batlle (2005).

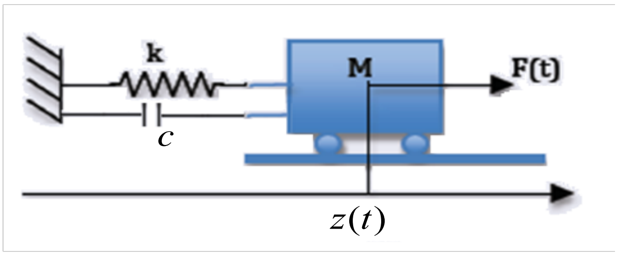
The following equation is derived using Newton's second law McCall (2010) [2] :

$$M \frac{d^2 z(t)}{dt^2} = F - kz(t) - c \frac{dz(t)}{dt} \tag{7}$$

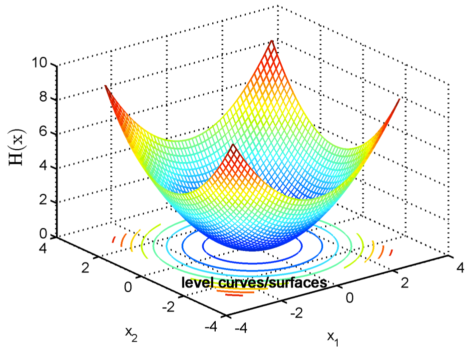
where :

- M is the mass of the body;
- F is the external force;
- k is the stiffness constant of the (linear) spring;
- c is the damping constant.

[2] This belongs to the so-called (generalized) Euler-Lagrange equations of classical mechanics Ortega *et al.* (1998); Van der Schaft (2000).



Hình 1. A mass-spring-damper system.



Hình 2. The Hamiltonian storage function $\mathcal{H}(x) = \alpha_1 x_1^2 + \alpha_2 x_2^2$ with $\alpha_1 = \alpha_2 = \frac{1}{2}$.

Let x be the vector consisting of the movement $z(t)$ and the momentum of the body $M\frac{dz(t)}{dt}$, i.e. $x = (x_1, x_2)^\top \equiv \left(z(t), M\frac{dz(t)}{dt}\right)^\top$, (7) can be rewritten as follows :

$$\begin{pmatrix} \frac{dx_1}{dt} \\ \frac{dx_2}{dt} \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -1 & -c \end{pmatrix} \begin{pmatrix} kx_1 \\ \frac{x_2}{M} \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} F \tag{8}$$

The system dynamics (8) leads to a pH representation (4) with :

$$J(x) = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \tag{9}$$

$$R(x) = \begin{pmatrix} 0 & 0 \\ 0 & c \end{pmatrix} \tag{10}$$

$$g(x) = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \ u = F \tag{11}$$

$$y = \frac{x_2}{M} \equiv \frac{dz(t)}{dt} \text{ (the velocity)} \tag{12}$$

and,

$$\mathcal{H}(x) = \frac{1}{2}kx_1^2 + \frac{1}{2}\frac{x_2^2}{M} \tag{13}$$

In this case, the Hamiltonian storage function $\mathcal{H}(x)$ (13) is equal to the total energy of the system, (i.e., it characterizes the amount of the elastic potential energy of the spring and the kinetic energy of the body, respectively). It therefore has the unit of energy. The damping matrix $R(x)$ (10) is symmetric and positive semi-definite.

Remark 1. As an analogy between mechanical and electrical systems Firestone (1933), it is worth noting that a second order ordinary differential equation of the series *RLC* circuit operated under a voltage source $V(t)$ can be written as follows :

$$L\frac{d^2 i(t)}{dt^2} + R\frac{di(t)}{dt} + \frac{1}{C}i(t) = \frac{dV(t)}{dt}$$

where $i(t)$ is the electric current. This is clearly equivalent to (7) in some sense.

For the sake of illustration, a geometric shape of the Hamiltonian storage function $\mathcal{H}(x)$ (13) is shown in Figure 2.

4. SOME FURTHER DISCUSSIONS

4.1 The proportional controller design

Let us state the following proposition.

Proposition 1. Under a zero state detectability condition and the boundedness from below of the Hamiltonian storage function $\mathcal{H}(x)$ by 0, it follows that an explicit proportional static output feedback law of the form,

$$u = -Ky \tag{14}$$

with $K = K^\top > 0$ a so-called damping injection gain, renders the controlled pH system dissipative and therefore asymptotically stabilized at the (singular) equilibrium $x^\star$.

Proof. From (6)(14), one obtains :

$$\frac{d\mathcal{H}(x)}{dt} \leq y^\top Ky < 0$$

The proof follows immediately by invoking La Salle's invariance principle Khalil (2002); Brogliato *et al.* (2007); Ortega *et al.* (2002). A complete version of the proof can be found in Hoang and Phan (2016). □

Remark 2. The convergence speed of the controlled system goes faster by increasing the controller gain K. Better performance of the controller can be proposed with the gain K derived from the Ziegler–Nichols tuning method.

Remark 3. Interestingly, the proportional feedback law u (14) can be considered in Figure 3 as control by simple interconnection where the controller here is so that

$$y_c = C(u_c) = Ku_c \tag{15}$$

It can be checked easily that

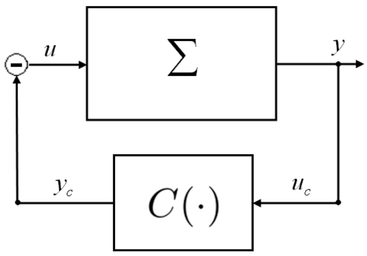
$$u_c(t)^\top y_c(t) + u(t)^\top y(t) = 0, \ \forall t \tag{16}$$

since $u_c(t) = y(t)$ and $u(t) = -y_c(t)$. The interconnection is therefore power continuous Ortega *et al.* (1998); Van der Schaft (2017); Batlle (2005).

Note also that the feedback law (14) may make the overall system worse when the Hamiltonian storage function $\mathcal{H}(x)$ (or also, the power) at any equilibrium except the trivial one $x^\star$ is nonzero Ortega *et al.* (2001). The situation is similar to that of the so-called dissipation obstacle Ortega *et al.* (2002).

4.2 Numerical simulations

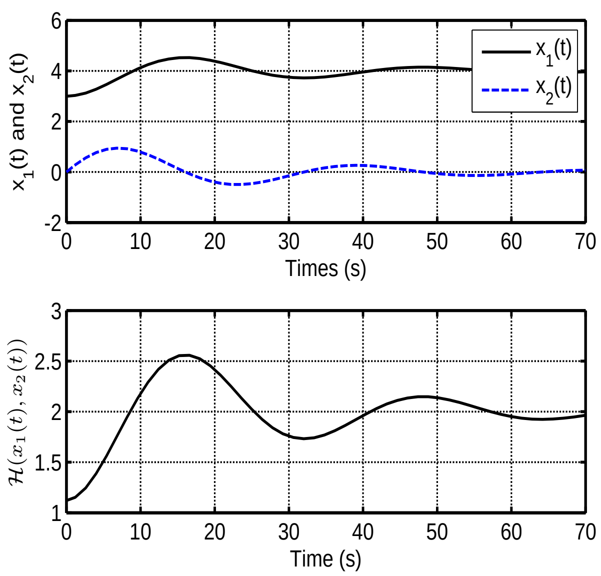
The simulations are carried out for the mass-spring-damper system using MATLAB & SIMULINK.



Hình 3. A standard negative feedback interconnection structure where Σ and $C(\cdot)$ are the plant and the controller, respectively.

The open loop SIMULINK model designed for the simulations is given in Figure A.1 of Appendix with $k = 0.25$ (N/m), $c = 0.5$ (N/(s.m)) and $M = 6.25$ (kg) (see also Longoria (2014))[3]. The input force of the system is a unit step, i.e., $u(t) = S(t)$ where $S(t)$ is the unit step function.

The initial conditions are chosen to be $x_1(t = 0) = x_{1,init} = 3$ and $x_2(t = 0) = x_{2,init} = 0$. Figure 4 shows the time evolutions of the states of the system and the storage function. It is shown that the storage function is bounded from below by a positive scalar. In other words, it is not equal to 0 since the states x_1 and/or x_2 converge to the nonzero values at steady state (i.e., $x_{1,ss} = \frac{F}{k}$ and $x_{2,ss} = 0$).



Hình 4. The states and storage function *w.r.t.* time.

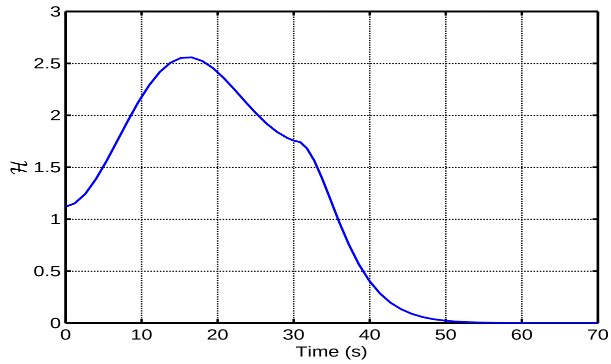
Now, we propose to stabilize the system at the natural equilibrium $x^\star = (x_1^\star, x_2^\star)^\top = (0, 0)^\top$. The control situation is that the system is operating normally with the unit step input, at time $t = t_1 > 0$, the unit step input is switched off and the proportional feedback law (14) is applied. The explicit expression of the manipulated input is then expressed as follows :

[3] It can be shown that the damping factor $\zeta := \frac{1}{2}\frac{c}{\sqrt{kM}}$ equals 0.2. The open loop system is therefore underdamped.

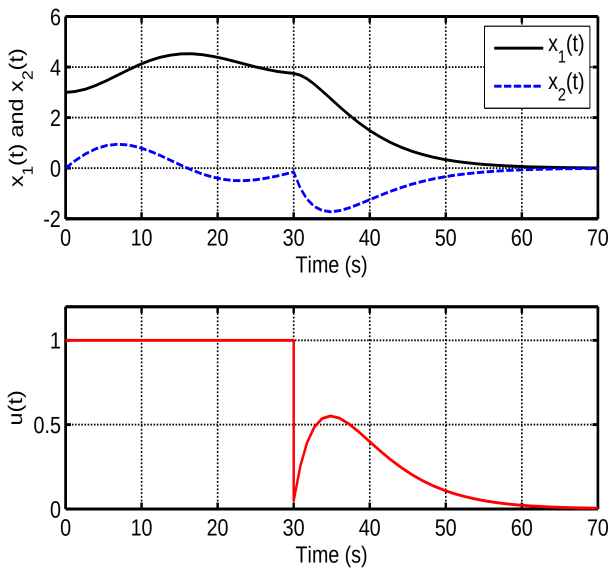
$$u(t) = \Big(S(t) - S(t - t_1)\Big) - K\frac{x_2}{M}S(t - t_1) \tag{17}$$

where equations (14) and (12) have been used. The closed loop SIMULINK model with the proportional feedback law is given in Figure A.2 of Appendix with $K = 2$ and $t_1 = 30$ (s).

Figure 5 shows that the controlled Hamiltonian storage function with the proportional feedback law (17) converges to 0 as $t \to +\infty$. As consequence, the global asymptotic convergence of the controlled states x to $x^\star$ is guaranteed as seen in Figure 6. Furthermore, the manipulated input u (17) is physically admissible in terms of amplitude and dynamics.



Hình 5. The controlled Hamiltonian storage function with the proportional feedback law.



Hình 6. The controlled states and manipulated input *w.r.t.* time with the proportional feedback law.

5. CONCLUSION

In this work, a port-based modeling of mass-spring-damper systems is reintroduced and leads to the so-called port-Hamiltonian representation. In this presentation, some structural properties such as interconnection and damping matrices and Hamiltonian storage function are explicitly shown. Interestingly, those terms have clear physical meaning. The feedback design and control scenarios proposed for the simulations are main contributions of the paper. As an important feature of the pH model, it remains now to extend this structure to biochemical reaction systems or multi-physics systems (see, e.g. Dörfler *et al.* (2009)).

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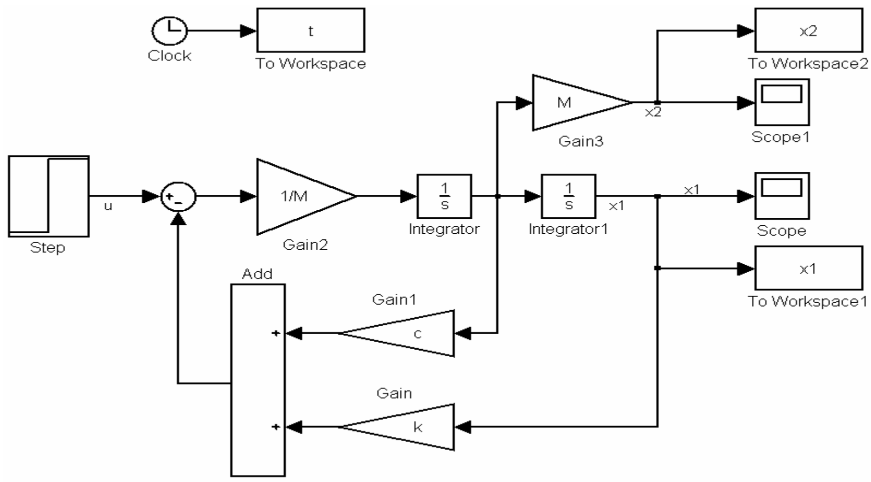
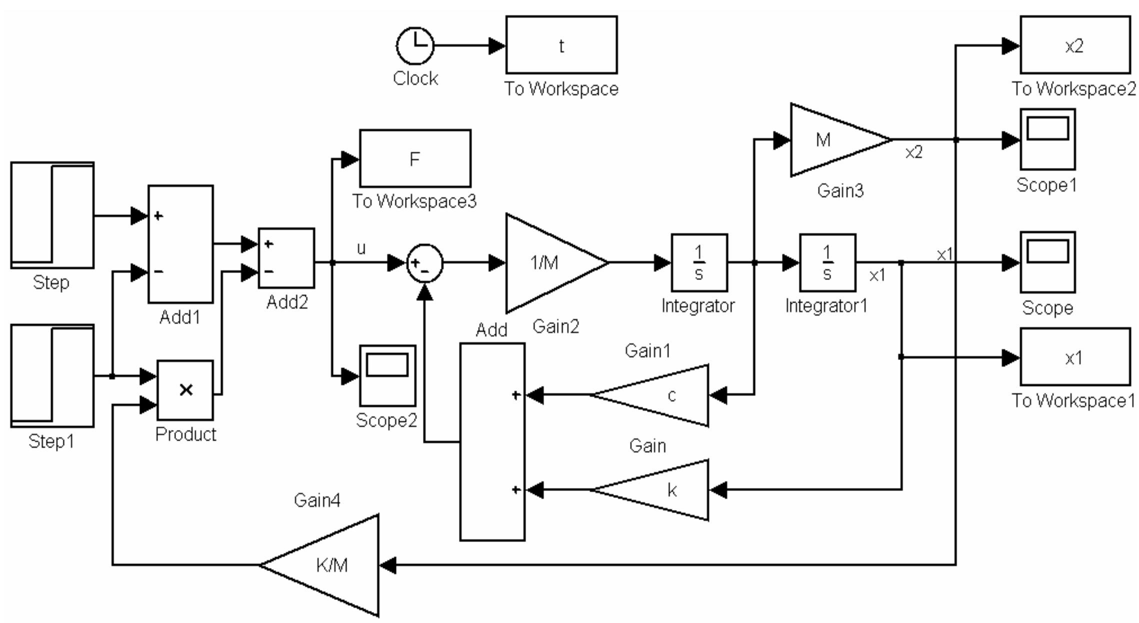
Phụ lục A. APPENDIX



Hình A.1. The open loop SIMULINK model.



Hình A.2. The closed loop SIMULINK model with the proportional feedback law.