Performance Analysis in Wireless Power Transfer System over Nakagami Fading Channels

Phu-Tuan Van, Hoai-Huong Nguyen Le, Mai-Duyen Nguyen Le, Dac-Binh Ha
Faculty of Electrical & Electronics Engineering, Duy Tan University, Da Nang, Vietnam
Email: phutuan87@gmail.com, nguyenlhoaihuong@dtu.edu.vn, maiduyen.l.nguyen@ieee.org, hadacbinh@duytan.edu.vn

***Abstract*—Wireless devices become useless in an absence of energy and become unable to contribute to the utility of the network as a group. Using a convenient and inadequate energy source for empowering the wireless devices is an attractive idea. However, Wireless Powered Communication (WPC) combined with information transmission depends on many issues. In this paper, we consider a single hop communication system in which one source receives energy from one power transfer station and uses the energy to transmit information to destination. The performance of considered WPC system is analyzed in terms of outage probability (P_{out}) and average symbol error probability (ASEP). The accuracy of the analyses verified by Monte-Carlo simulations validate that energy harvesting time coefficient, modulation order, shape parameter of Nakagami channels, distance from source to destination and are important ones to control P_{out} and ASEP of WPC.**

***Index Terms*—Wireless Powered Communication, outage probability, ASEP, M-PSK , Nakagami-m.**

I. INTRODUCTION

Nowadays, a big issue facing in wireless network is a battery lifetime. Actually, the resource-constrained nature of wireless devices, especially the battery lifetime is limited their operating cycles. In case of emergency without manually charging battery, premature battery exhaustion can be occurred while the wireless devices are transferring the information to other wireless devices. Due to being concerned about this scenario, the concept of a green radio communication (GRC) network is offered and quickly becomes an attractive research topic in both industry and academia. Nowadays, the techniques are applied for the green communication network named Simultaneous Wireless Information and Power Transfer (SWIPT), Energy Harvesting (EH), and Wireless Powered Communication (WPC). SWIPT could be efficiently built to transmit information and energy simultaneously using the same waveform [1], [2]. Derrick Wing Kwan Ng et. al. [3] introduced a multi-objective optimization problem formulation for the power allocation algorithm design for SWIPT. In addition, EH is a technique to collect energy from the surrounding environment for prolonging the lifetime of a wireless network [4]–[8]. In [9], a cooperative system in which EH nodes volunteer used as AF relays whenever they have sufficient energy for transmission over frequency-flat, block-fading Rayleigh channels is considered. The closed form expressions for the symbol error rate (SER) of the system and the asymptotic energy savings at the source from the use of EH relays are derived. The analysis show that the energy usage at an EH relay depends not only on the relay's energy harvesting process, but also on its transmit power setting and the other relays in the system. Another green communication technique is WPC [10] where the wireless devices are powered by dedicated wireless power transmitters to provide continuous and stable microwave energy over the air. Based on WPC technique, a wireless powered communication network (WPCN) is offered to supply wireless communication devices with a controllable power in the different physical conditions. WPCN is believed that it achieves more high-efficiency than a traditional battery-powered communication. In addition to, WPCN provides greater flexibility at lower cost.

Wireless channel fading and interference are major challenges needed to pay attention in any wireless communication. Many models are offered to describe channel fading such as Rayleigh, Rician, Nakagami, etc. In practice, Nakagami fading model is more general than Rayleigh, Rician fading model in a wireless channel fading. In [11], the authors have derived closed-form expressions for the average symbol error probability (ASEP) for the dual-hop relaying system that all devices are equipped with multiple antennas with dissimilar Nakagami fading channel. In [12], uncorrelated-Nakagami is considered, where ASEP is derived for the single hop system that all devices are equipped with single antenna. Actually, there were no published studies for analyzing the performance of WPCN over Nakagami fading channel.

In this paper, we focus on a single hop WPCN over Nakagami fading channel which one source node receives energy from one power transfer node and uses the energy to transmit information to one destination node that all of them are equipped with single antenna in term of two criterions: P_{out} and ASEP. In addition, we also analyze the performance of the considered system in various system parameters, such as energy harvesting time, energy harvesting efficiency, and relay location, etc.

The paper is organized as follows: Section II presents the system and channel model. Performance of the considered system is analyzed in Section III. In Section IV, we show the numerical results. We conclude our work in Section V.

II. SYSTEM AND CHANNEL MODEL

We consider a WPC system illustrated in Fig 1. The network consists of one power transfer station, one source and one destination denoted by $\mathbb{E}$, $\mathbb{S}$ and $\mathbb{D}$, respectively. $\mathbb{S}$ uses time-swiching protocol to collects the energy from $\mathbb{E}$ and uses this energy to transmit the its information to the $\mathbb{D}$. Throughout this paper, the following set of assumptions are considered: 1) The processing power required by the transceiver at the $\mathbb{S}$ is negligible as compared to the power used for signal transmission. 2) The channel is constant over the block time T and independent and identically distributed in each time blocks. 3) The channel of $\mathbb{E}-\mathbb{S}$ link and $\mathbb{S}-\mathbb{D}$ link (the link from power transfer station to source and from source to destination, respectively) are Nakagami fading channels with these shape parameters are m and n, respectively. 4) All devices are equipped with one transmit and one receive antennas.

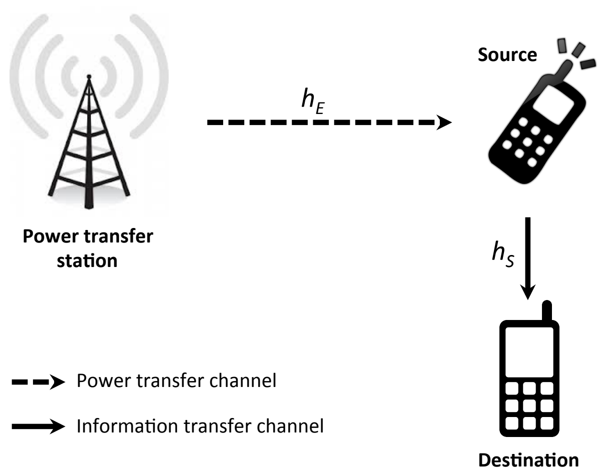

Figure 1. System model of WPC

First, the source harvests the energy E_S from the power transfer station in duration αT and uses this energy for signal transmission in duration $(1-\alpha)T$ with transmit power P_S. E_S and P_S have following forms [13]

$$E_S = \frac{\eta P_0 \left|h_E\right|^2 \alpha T}{d_E^\theta}, \tag{1}$$

$$P_S = \frac{E_S}{(1-\alpha)T} = \frac{\eta P_0 \alpha \left|h_E\right|^2}{(1-\alpha)d_E^\theta}, \tag{2}$$

where P_0 is the transfer power of $\mathbb{E}$; $0 \le \eta \le 1$ is the energy conversion efficiency which depends on the rectification process and the energy harvesting circuitry; $\mathbb{S}$ harvests energy from the signal of $\mathbb{E}$ in every block times T with the energy harvesting time coefficient $0 \le \alpha \le 1$; $|h_E|^2$ is the channel power gain of the $\mathbb{E}-\mathbb{S}$ link ; d_E is the distance from power transmit station to source; θ is the path loss exponent;

In duration $(1-\alpha)\,T$, the source transmits its signal $x(t)$ to the destination, the received signal $y\,(t)$ at $\mathbb{D}$ is given by

$$y(t) = \frac{\sqrt{P_S} h_S}{d_S^\theta} x(t) + n_D, \tag{3}$$

where d_S is the distance from source to destination and n_D is white complex Gaussian noise, $n_D \sim \mathcal{CN}\,(0,1)$.

The instantaneous received SNR at the destination is given by

$$\gamma_D \quad = \quad \frac{\eta P_0 \alpha \left|h_E\right|^2 \left|h_S\right|^2}{(1-\alpha)d_E^\theta d_S^\theta} = a\gamma_1\gamma_2, \tag{4}$$

where $a = \dfrac{\eta P_0 \alpha}{(1-\alpha)d_E^\theta d_S^\theta}$, $\gamma_1 = |h_E|^2$ and $\gamma_2 = |h_S|^2$.

The probability density function (PDF) of RV γ_1 is is given by

$$f_{\gamma_1}(x) = \frac{m^m x^{m-1}}{\Gamma(m)\bar{\gamma}_1^m} e^{-\frac{mx}{\bar{\gamma}_1}}, \tag{5}$$

where $\bar{\gamma}_1 = \mathbf{E}\left[|h_E|^2\right]$; $\mathbf{E}\,[.]$ is expectation operator.

The cumulative density function (CDF) of RV γ_1 is given by

$$F_{\gamma_1}(x) \quad = \quad 1 - e^{-\frac{mx}{\bar{\gamma}_1}} \sum_{k=0}^{m-1} \left[\frac{1}{k!}\left(\frac{mx}{\bar{\gamma}_1}\right)^k\right]. \tag{6}$$

Similarly, the PDF and CDF of RV γ_2 are given by

$$f_{\gamma_2}(x) = \frac{n^n x^{n-1}}{\Gamma(n)\bar{\gamma}_2^n} e^{-\frac{nx}{\bar{\gamma}_2}}, \tag{7}$$

$$F_{\gamma_2}(x) = 1 - e^{-\frac{nx}{\bar{\gamma}_2}} \sum_{k=0}^{n-1} \left[\frac{1}{k!}\left(\frac{nx}{\bar{\gamma}_2}\right)^k\right], \tag{8}$$

where $\bar{\gamma}_2 = \mathbf{E}\left[|h_S|^2\right]$.

III. PERFORMANCE ANALYSIS

A. Outage Probability (OP)

Outage probability is an important performance metric that is generally used to characterize a wireless communication system. It is defined as the probability that the instantaneous SNR at the destination, γ_D, falls below a predetermined threshold γ_{Th}, given by

$$\begin{aligned} P_{out} &= F_{\gamma_D}\left(\gamma_{Th}\right) = Pr\left(a\gamma_1\gamma_2 < \gamma_{Th}\right) \\ &= \int_0^\infty Pr\left[\gamma_1 < \frac{\gamma_{Th}}{a\gamma_2} \mid_{\gamma_2}\right] f_{\gamma_2}\left(\gamma_2\right) d\gamma_2, \end{aligned} \tag{9}$$

where $F_{\gamma_D}\left(\gamma_{Th}\right)$ is CDF of the instantaneous SNR at the destination.

Theorem 1. *The system OP is follow as*

$$P_{out} = 1 - \sum_{k=0}^{m-1} \frac{2}{k!\Gamma\left(n\right)} \left(\lambda\gamma_{Th}\right)^{\frac{n+k}{2}} \mathcal{K}_{n-k}\left(2\sqrt{\lambda\gamma_{Th}}\right), \tag{10}$$

where $\lambda = \frac{mn}{\bar{\gamma}_1\bar{\gamma}_2 a}$

Proof: A proof of this theorem is provided in the Appendix A. ■

B. Average Symbol Error Probability (ASEP)

ASEP is an another important system performance measure that is useful for system designers to evaluate a wireless communication system. The ASEP of communication link over fading channels is given by

$$P_s(\gamma) = \int_0^\infty P_{s|\gamma}(\gamma) f_D(\gamma) d\gamma, \tag{11}$$

where $P_{s|\gamma}(\gamma) = \rho Q\left(\sqrt{\mu\gamma}\right)$ with $Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-t^2/2} dt$ is the Gaussian Q-function; ρ and μ are constants specific to modulation type. For example, with BPSK modulation $\rho = 1$ and $\mu = 2$, with QPSK modulation $\rho = 2$ and $\mu = 1$, etc.

Theorem 2. *The system ASEP is follow as*

$$P_s(\gamma) = \frac{\rho}{2} - \sum_{k=0}^{m-1} \frac{\rho \left(\frac{\lambda}{\mu}\right)^{\frac{n+k}{2}} 2^{\frac{n+k}{2}-1}}{\sqrt{2\pi} k! \Gamma(n)} \Gamma\left(k + \frac{1}{2}\right) \Gamma\left(n + \frac{1}{2}\right) \times e^{-\frac{\lambda}{\mu}} \mathcal{W}_{-\frac{n+k}{2}, \frac{n-k}{2}} \left(2\frac{\lambda}{\mu}\right) dt, \tag{12}$$

Proof: A proof of this theorem is provided in the Appendix B. ∎

IV. NUMERICAL RESULTS AND DISCUSSION

In this section, we discuss some results based on the theoretical analysis and Monte-Carlo simulations of the outage probability and the Average Symbol Error Probability of considered system.

A. Outage Probability (P_{out})

We investigate the effects of γ_{Th}, the shape parameters and α on the outage probability by using the analytical equation (10) and simulation results. In Fig. 2, we examine the impact of γ_{Th} on scheme. With the fixed γ_{Th}, the harvest energy of $\mathbb{S}$ will follow with (1). We assume the channel power gain is the constant at each block time, thus when we increase P_0, $\mathbb{S}$ will harvest more energy. Therefore, the signal to noise ratio at $\mathbb{D}$ increases so that P_{out} decreases. On the other hand, with the fixed P_0, when we increase γ_{Th}, P_{out} will increase.

Fig. 3 shows the effect of shape parameter on P_{out}. Because the role of m and n are the same in γ_D, we have only analyse the impact of m. Consider with high value of P_0, when we increase m , P_{out} will decreases slightly. On the orther hand, with low value of P_0, when we increase m, P_{out} will increase moderately.

In order to obtain more harvesting energy at $\mathbb{S}$, we can increase the energy harvesting time coefficient (α) or the energy conversion efficiency (η), etc. Fig. 4 shows the impact of α on P_{out}. With the high value of α, the $\mathbb{S}$ will spent more time for haverting energy. Therefore, $\mathbb{S}$ receives more energy, the γ_D at $\mathbb{D}$ increases and the P_{out} of considered system rises. However, it is also the cause for decreasing the capacity of data transmission because $\mathbb{S}$ uses less time for transmitting information. On the orther hand, with the low value of α, $\mathbb{S}$ will spent more time for data transmission than harverting energy so the P_{out} decreases but the capacity of data transmission increases.

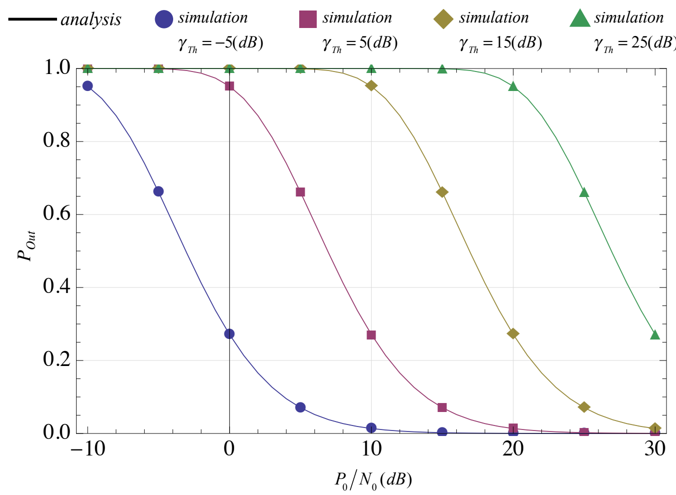

Figure 2. Effect of γ_{Th} on P_{out} with $m = n = 2, \eta = 1, \alpha = 0.5, d_S = d_M = 1$ and $\theta = 3$

B. Average Symbol Error Probability (ASEP)

We analysis ASEP of investigated system using M-PSK modulation. Fig 5 shows the ASEP results with variety of values of M. The ASEP will be less when we use higher modulation order. With M is greater than 2, we approximate the average symbol error propability for a given value of γ_D by using Gaussian Q-function, thus the simulation and analysic results have a little differents.

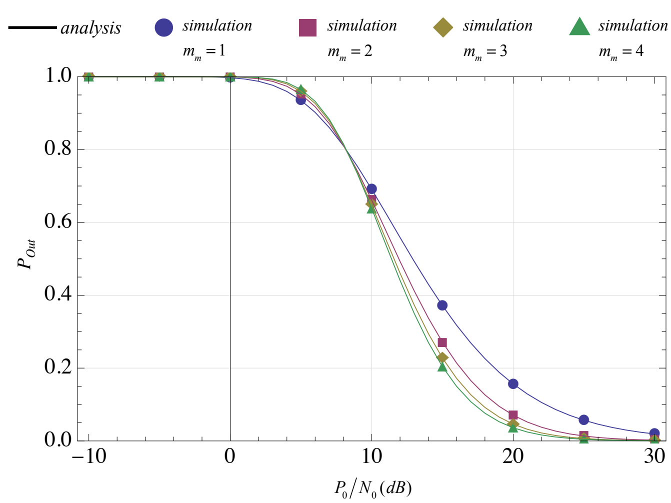

Figure 3. Effect of m on P_{out} with $\gamma_{Th} = 10dB, n = 2, \eta = 1, \alpha = 0.5, d_S = d_M = 1$ and $\theta = 3$

Similarly, we investigate the effect of m on the ASEP. In Fig 6, consider with the high value of P_0, the ASEP at $\mathbb{D}$ when we use a high value of m is better than when we use

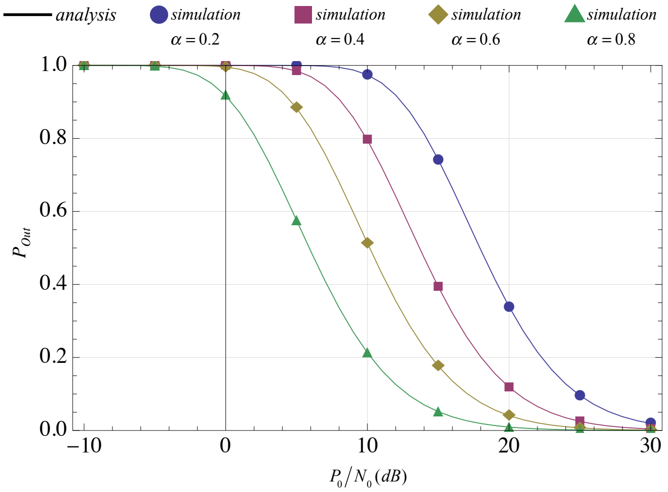

Figure 4. Effect of α on P_{out} with $\gamma_{Th} = 10dB, m = n = 2, \eta = 1, d_S = d_M = 1$ and $\theta = 3$

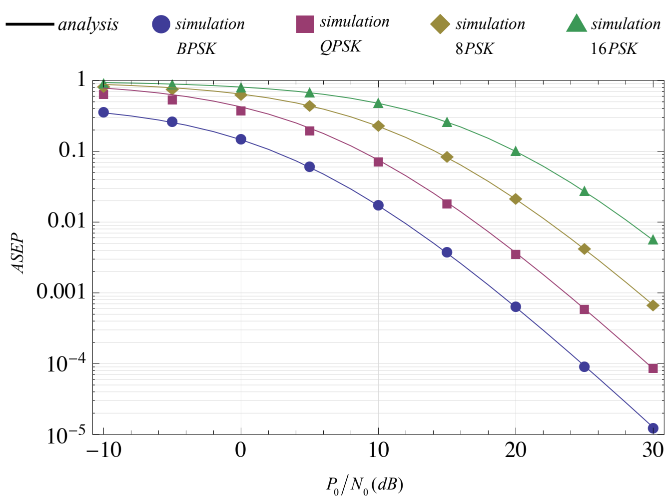

Figure 5. Effect of modulation type on ASEP with $m = n = 2, \eta = 1, \alpha = 0.5, d_S = d_M = 1$ and $\theta = 3$

a low value of m. This assessment is resonable because the propability density of γ_D when using higher shape parameter is more focus than the contrary case.

Fig 7 shows the effect of α on ASEP of investigate system. When $\mathbb{S}$ spents more time for harvesting energy from $\mathbb{P}$, $\mathbb{S}$ will transmit its information with higher power sothat ASEP of system in this case is better than when $\mathbb{S}$ spent more time for data tramission.

V. CONCLUSION

In this paper, a single hop wireless powered communication system over Nakagami fading channel is analysed with assume that the source uses time-switch protocol for energy harvesting and information transmission. The exact closed form expression of P_{out} and approximation expression of ASEP have been derived. From these analysis results we can evaluate the performance of considered system in the influence of γ_{Th}, modulation order, α, and shape parameter of Nakagami channel. These analytical derivations have been validated by Monte-Carlo simulations.

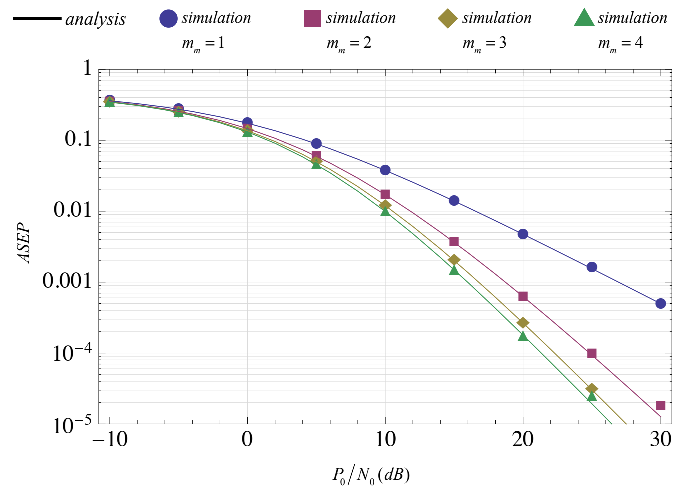

Figure 6. Effect of m on ASEP with $M = 2, n = 2, \eta = 1, \alpha = 0.5, d_S = d_M = 1$ and $\theta = 3$

APPENDIX A
PROOF OF THEOREM 1

we have used the following relation into (9) to obtain (10)

$$\int_0^\infty t^{\nu-1} e^{-\frac{\beta}{t}-\alpha t} dt = 2\left(\frac{\beta}{\alpha}\right)^{\nu/2} \mathcal{K}_\nu\left(2\sqrt{\beta\alpha}\right), \quad (13)$$

where β, α are positive real values and $\mathcal{K}_\nu(.)$ is the modified Bessel function of the second kind and ν^{th} order.

$$\begin{aligned} P_{out} &= \int_0^\infty Pr\left[\gamma_1 < \frac{\gamma_{Th}}{a\gamma_2} |_{\gamma_2}\right] f_{\gamma_2}(\gamma_2) d\gamma_2 \\ &= \int_0^\infty F_{\gamma_1}\left(\frac{\gamma_{Th}}{a\gamma_2}\right) f_{\gamma_2}(\gamma_2) d\gamma_2 \\ &= \int_0^\infty \left(1 - e^{-\frac{m\gamma_{Th}}{\bar{\gamma}_1 a\gamma_2}} \sum_{k=0}^{m-1} \frac{1}{k!}\left(\frac{m\gamma_{Th}}{\bar{\gamma}_1 a\gamma_2}\right)^k\right) \\ &\quad \times \frac{n^n \gamma_2^{n-1}}{\Gamma(n)\bar{\gamma}_2^n} e^{-\frac{n\gamma_2}{\bar{\gamma}_2}} d\gamma_2 \\ &= 1 - \sum_{k=0}^{m-1} \int_0^\infty \frac{n^n m^k \gamma_{Th}^k \gamma_2^{n-1-k}}{k!(\bar{\gamma}_1 a)^k \Gamma(n)\bar{\gamma}_2^n} e^{-\frac{m\gamma_{Th}}{\bar{\gamma}_1 a\gamma_2} - \frac{n\gamma_2}{\bar{\gamma}_2}} d\gamma_2 \\ &= 1 - \sum_{k=0}^{m-1} \frac{2}{k!\Gamma(n)} (\lambda\gamma_{Th})^{\frac{n+k}{2}} \mathcal{K}_{n-k}\left(2\sqrt{\lambda\gamma_{Th}}\right). \end{aligned}$$

APPENDIX B
PROOF OF THEOREM 2

According to [14], we can rewrite (11) as follows

$$P_s(\gamma) = \frac{\rho}{2\sqrt{2\pi}} \int_0^\infty F_{\gamma_D}\left(\frac{t}{\mu}\right) e^{-t/2} t^{-1/2} dt. \quad (14)$$

From (14), we use two following equations (3.361.1) and (6.643.3) in [15] to yields (12)

$$\int_0^\infty \frac{e^{-qt}}{\sqrt{t}} dt = \sqrt{\frac{\pi}{q}}, \quad (15)$$

$$\int_0^\infty t^{\upsilon-\frac{1}{2}} e^{-\alpha t} \mathcal{K}_{2\nu}\left(2\beta\sqrt{t}\right) dt = \frac{\alpha^{-\mu}}{2\beta} exp\left(\frac{\beta^2}{2\alpha}\right)$$
$$\times \mathcal{W}_{-\upsilon,\nu}\left(\frac{\beta^2}{\alpha}\right) \Gamma\left(\upsilon - \nu + \frac{1}{2}\right) \Gamma\left(\upsilon + \nu + \frac{1}{2}\right), \quad (16)$$

where $t = \sqrt{\mu\gamma}$ and $\mathcal{W}(.)$ is the Whittaker function.

$$\begin{aligned} P_s(e) &= \frac{\rho}{2\sqrt{2\pi}} \int_0^\infty F_{\gamma_D}\left(\frac{t}{\mu}\right) e^{-t/2} t^{-1/2} \\ &= dt \frac{\rho}{2\sqrt{2\pi}} \int_0^\infty e^{-t/2} t^{-1/2} \\ &\times \left(1 - \sum_{k=0}^{m-1} \frac{2}{k!\Gamma(n)} \left(\frac{\lambda}{\mu} t\right)^{\frac{n+k}{2}} \mathcal{K}_{n-k}\left(2\sqrt{\frac{\lambda}{\mu} t}\right)\right) dt \\ &= \frac{\rho}{2\sqrt{2\pi}} \int_0^\infty e^{-t/2} t^{-1/2} dt - \sum_{k=0}^{m-1} \frac{\rho\left(\frac{\lambda}{\mu}\right)^{\frac{n+k}{2}}}{\sqrt{2\pi} k! \Gamma(n)} \\ &\times \int_0^\infty t^{\frac{n+k-1}{2}} e^{-t/2} \mathcal{K}_{n-k}\left(2\sqrt{\frac{\lambda}{\mu} t}\right) dt \\ &= \frac{\rho}{2} - \sum_{k=0}^{m-1} \frac{\rho\left(\frac{\lambda}{\mu}\right)^{\frac{n+k}{2}} 2^{\frac{n+k}{2}-1}}{\sqrt{2\pi} k! \Gamma(n)} \Gamma\left(k + \frac{1}{2}\right) \\ &\times \Gamma\left(n + \frac{1}{2}\right) e^{-\frac{\lambda}{\mu}} \mathcal{W}_{-\frac{n+k}{2}, \frac{n-k}{2}}\left(2\frac{\lambda}{\mu}\right) dt. \end{aligned}$$

References

[1] R. Zhang and C. K. Ho, "MIMO broadcasting for simultaneous wireless information and power transfer," *Wireless Communications, IEEE Transactions on*, vol. 12, no. 5, pp. 1989–2001, 2013.

[2] X. Zhou, R. Zhang, and C. K. Ho, "Wireless information and power transfer: architecture design and rate-energy tradeoff," in *Global Communications Conference (GLOBECOM), 2012 IEEE*. IEEE, 2012, pp. 3982–3987.

[3] D. W. K. Ng, L. Xiang, and R. Schober, "Multi-objective beamforming for secure communication in systems with wireless information and power transfer," in *Personal Indoor and Mobile Radio Communications (PIMRC), 2013 IEEE 24th International Symposium on*. IEEE, 2013, pp. 7–12.

[4] V. Raghunathan, S. Ganeriwal, and M. Srivastava, "Emerging techniques for long lived wireless sensor networks," *Communications Magazine, IEEE*, vol. 44, no. 4, pp. 108–114, 2006.

[5] P. Popovski, A. M. Fouladgar, and O. Simeone, "Interactive joint transfer of energy and information," *Communications, IEEE Transactions on*, vol. 61, no. 5, pp. 2086–2097, 2013.

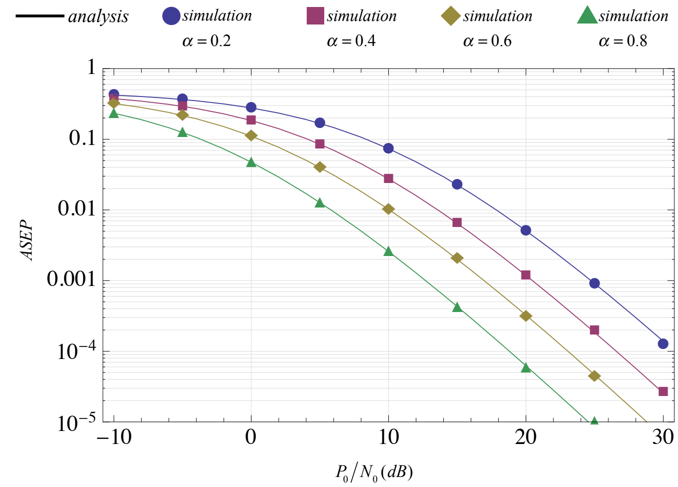

Figure 7. Effect of α on ASEP with $M = 2, m = n = 2, \eta = 1, d_S = d_M = 1$ and $\theta = 3$

[6] J. Xu and R. Zhang, "Throughput optimal policies for energy harvesting wireless transmitters with non-ideal circuit power," *Selected Areas in Communications, IEEE Journal on*, vol. 32, no. 2, pp. 322–332, 2014.

[7] S. Luo, R. Zhang, and T. J. Lim, "Optimal save-then-transmit protocol for energy harvesting wireless transmitters," *Wireless Communications, IEEE Transactions on*, vol. 12, no. 3, pp. 1196–1207, 2013.

[8] L. Liu, R. Zhang, and K.-C. Chua, "Wireless information transfer with opportunistic energy harvesting," *Wireless Communications, IEEE Transactions on*, vol. 12, no. 1, pp. 288–300, 2013.

[9] B. Medepally and N. B. Mehta, "Voluntary energy harvesting relays and selection in cooperative wireless networks," *Wireless Communications, IEEE Transactions on*, vol. 9, no. 11, pp. 3543–3553, 2010.

[10] N. Shinohara, "Power without wires," *Microwave Magazine, IEEE*, vol. 12, no. 7, pp. S64–S73, 2011.

[11] G. C. Alexandropoulos, P. C. Sofotasios, K. Ho-Van, and S. Freear, "Symbol error probability of DF relay selection over arbitrary nakagami-fading channels," *Journal of Engineering*, vol. 2013, 2013.

[12] N. Sagias, "On the ASEP of decode and forward dual hop networks with pilot symbol assisted MPSK," 2014.

[13] A. A. Nasir, X. Zhou, S. Durrani, and R. A. Kennedy, "Relaying protocols for wireless energy harvesting and information processing," *Wireless Communications, IEEE Transactions on*, vol. 12, no. 7, pp. 3622–3636, 2013.

[14] H. A. Suraweera, G. K. Karagiannidis, and P. J. Smith, "Performance analysis of the dual-hop asymmetric fading channel," *Wireless Communications, IEEE Transactions on*, vol. 8, no. 6, pp. 2783–2788, 2009.

[15] I. Gradshteyn and I. Ryzhik, *Table of integrals, series, and products*. Academic Press, 2007.